INTERFERENCE MANAGEMENT USING GRASSMANNIAN PACKING

A THESIS

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THESIS CERTIFICATE

This is to certify that the thesis titled **INTERFERENCE MANAGEMENT USING GRASSMANNIAN PACKING**, submitted by **K.V.Sidharth (EE09B035)**, to the Indian Institute of Technology Madras, Chennai for the award of the degree of **Bachelor of Technology**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Precoding; Grassmannian Packing; Log-Likelihood Ratio; Gray-coding Reduced Log-Likelihood Ratios

Evolution of technology from 2G to 3G to 4G has created rapid increase in the data rates in mobile communication. The latest 4G standards such as Long Term Evolution (LTE) are aiming to provide data rates of up to 100 MB/s in downlink and 50 MB/s in uplink. In order to provide increasing data rates per user, individual users are being allocated higher bandwidth. In 2G technologies such as Global System for Mobile (GSM), the per user allocated bandwidth is 200Khz, whereas in LTE Release 10, per user allocated bandwidth can go up to a maximum of 20MHz. However, the total available spectrum is limited and very expensive for licensing.

Because of the effects of increase in number of mobile subscribers and increasing per user bandwidth allocation, the frequency spectrum cannot be split up to avoid interference. At the same time, reuse-1 gives very high error rates even at low interference power. Hence, reuse schemes between reuse-1 and reuse-3 are used so that interference as well as error rates are kept in check.

So, precoders are used that reduce the effect of interference from other interfering transmitters. For having rates greater than $1/L$ in a L user interference channel, use of vectors that have been designed as the optimal Grassmannian packing in a K dimensional space is proposed. These vectors are combined together to get precoder matrices for every transmitter.

In second part of my thesis, the Log-Likelihood Ratio for each bit of Gray-coded BPSK, QPSK, 16QAM and 64QAM is computed. Due to Gray-coding, these ratios can be reduced to simpler forms. This decreases the computation time of LLRs by a big factor, thus helping in faster decoding. These simplified expressions are the Reduced Log-Likelihood Ratios (RLLR).

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ABBREVIATIONS

MHz	Mega Hertz
LTE	Long Term Evolution
BER	Bit Error Rate
AWGN	Additive White Gaussian Noise
ICBM	Interference Cancelling Block Modulation
QPSK	Quadrature Phase Shift Keying
QAM	Quadrature Amplitude Modulation
SNR	Signal to Noise Ratio
SIR	Signal to Interference Ratio
BPSK	Binary Phase Shift Keying
LLR	Log Likelihood Ratio
dB	Decibel
RLLR	Reduced Log Likelihood Ratio

NOTATIONS

A	Matrix
v	Vector
T_i	i^{th} Transmitter
R_i	i^{th} Receiver
$\mathbb{R}$	Set of all real numbers
A^T	Transpose of a Matrix
A^{-1}	Inverse of a Matrix
$\|.\|$	Norm of a vector
$\|\|.\|\|$	Norm of a vector
Log	Natural Logarithm
$\rightarrow$	Implies
$A * B$	Matrix multiplication of A and B
$P(A\|B)$	Probability of A given B

CHAPTER 1

INTRODUCTION

India has one of the fastest growing telecom network in the world due to the developing status of the country and its high population. Mobile telephony was first introduced in India in 1995. Since then, the number of mobile phone subscribers has been tremendously increasing. The total number of mobile phone subscribers has reached 929.37 million as of May 2012 [1]. The mobile tele-density has increased to 76.68% in May 2012 [1].

With increasing number of subscribers and limited bandwidth for use, there is always a need for increasing rates. The idea of providing coverage to mobile phones over the whole of geographical area was first proposed by Douglas Ring in 1947 [2]. The geographical area is divided into hexagonal regions called cells. A single basestation covers the whole of hexagonal region. When the neighbouring basestations of other cells also transmit on the same frequency band, the users at the cell boundaries receive transmissions from other basestations as well. But, the user only wants to receive the signal transmitted by his basestation. Hence, the signal transmitted by other basestations act as interference to the desired data sent to the user. To avoid interference, reuse-n schemes were used, n generally being 3 or 7, where frequency band was divided into n parts to be used individually by each interfering basestation as shown in Figure (1.1).

As technology is moving from 2G to 3G to 4G, the data rates have been going up. The latest 4G standards such as Long Term Evolution (LTE) are aiming to provide data rates of up to 100 MB/s in downlink and 50 MB/s in uplink. In order to provide increasing data rates per user, individual users are being allocated higher bandwidth. In 2G technologies such as Global System for Mobile (GSM), the per user allocated bandwidth is 200Khz, whereas in LTE Release 10, per user allocated bandwidth can go up to a maximum of 20MHz. However, the total available spectrum is limited and expensive for licensing.

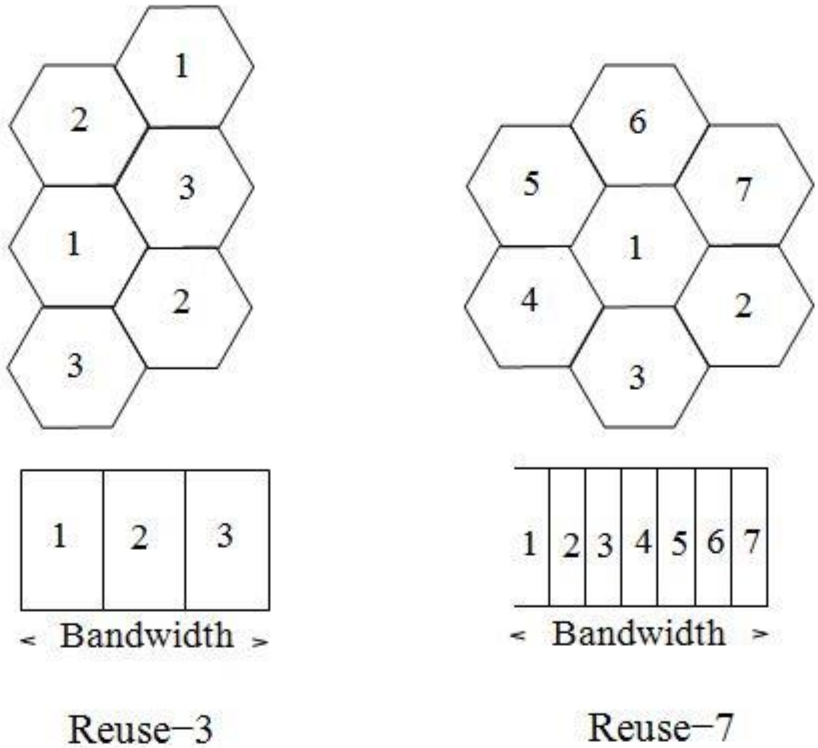


Figure 1.1: Reuse-3 and Reuse-7 schemes [3]

Because of the effects of increase in number of mobile subscribers and increasing per user bandwidth allocation, the frequency spectrum cannot be split up to avoid interference. At the same time, reuse-1 gives very high error rates even at low interference power. Hence, techniques which have effective reuse rates between reuse-1 and reuse-3 are used so that interference as well as error rates are kept in check.

To reduce the effect of interference from one basestation on other, the help of optimal Grassmannian packing is taken and precoders are designed using these packing. The precoders at each basestation are designed such that every basestation uses all the resources and also such that the effect of interference from other basestations is nullified, thus keeping the effective reuse rate above reuse-3 and also reducing the interference effect at the same time.

CHAPTER 2

DESIGN OF PRECODER MATRICES FOR INTERFERENCE CHANNELS

Consider a general scenario of L-user interference channel. Normalising the amplitude of the desired signal received to be 1 at R_i, the amplitude of the L-1 interfering signals is taken to be having an average amplitude of $\sqrt{\alpha_j}$, $j \neq i$. Each of the transmitters T_i sends

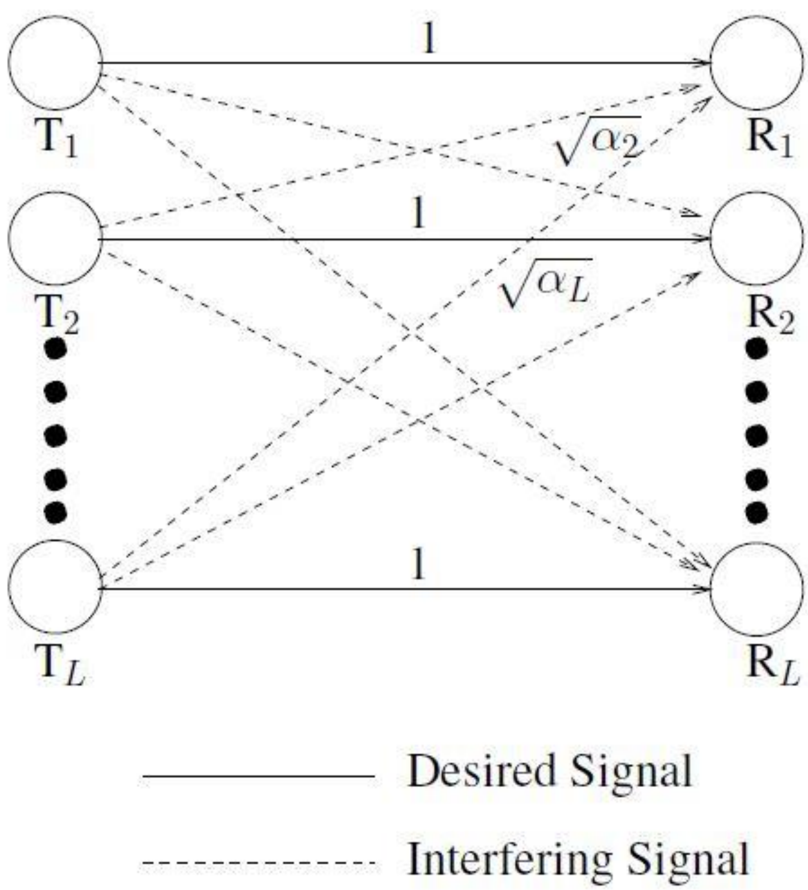


Figure 2.1: Interference model for a general L user case

data to be received by the corresponding R_i, but the desired signal experiences interference from all other transmitters $T_j, j \neq i$.

2.1 Quasi-Orthogonal Precoders

In a general L user interference case, the precoders are designed such that each signal has least possible effect on the others. Suppose 1 symbol has to be sent by every user in the L user interference case using K resource blocks, each user is required to have a $K \times 1$ precoder matrix. If $K \geq L$, then every transmitter has at least one resource block exclusively for itself. Every transmitter can transmit in its own resource block and can totally avoid interference.

If $K < L$, then each user's $K \times 1$ matrix should be designed to avoid interference from others. Let the L vectors be $v_1, v_2, \ldots, v_L$. The combined precoder matrix is given by:

$$Q^{K \times L} = [v_1\ v_2\ v_3 \ldots v_L] \quad (2.1)$$

Consider the design where there are L vectors in a K dimensional space such that the minimum distance **d**, as defined below, between any two pair of vectors is maximised. The **d** is the measure of interference effect of one vector on other.

$$\boldsymbol{d} = min(\|v_i - v_j\|, \|v_i + v_j\|)\,, i \neq j \quad (2.2)$$

To maintain symmetry, all the vectors are assumed to be of the same length and all the vectors are also assumed to have real entries. Taking these extra conditions into account, the max-min problem[1] is simplified.

[1] Here, the max-min problem refers to an optimisation problem where the set of minima of numerous sets has to be maximised.

The optimisation problem boils down to:

$$v_1, v_2, \dots, v_L: maximise\left(min\left(\|v_i - v_j\|, \|v_i + v_j\|\right), i \neq j\right) \tag{2.3}$$

$$\rightarrow v_1, v_2, \dots, v_L: maximise\left(min\left(\|v_i - v_j\|^2, \|v_i + v_j\|^2\right), i \neq j\right) \tag{2.4}$$

$$\rightarrow v_1, v_2, \dots, v_L: maximise\left(min\left(-2|v_i \cdot v_j|, i \neq j\right)\right) \tag{2.5}$$

$$\rightarrow v_1, v_2, \dots, v_L: minimise\left(max\left(|v_i \cdot v_j|, i \neq j\right)\right) \tag{2.6}$$

The angle between two vectors v_i and v_j is defined by:

$$Cos\theta_{ij} = \frac{v_i \cdot v_j}{\|v_i\|\|v_j\|} \tag{2.7}$$

Where $v_i \cdot v_j = \sum_{k=1}^{k=L} v_{ki} v_{kj}$, (2.8)

v_{ki} being the component of vector v_i in the k^{th} dimension. Hence, the condition in Equation (2.6) simplifies to:

$$v_1, v_2, \dots, v_L: minimise\left(max\left(|Cos\theta_{ij}|, i \neq j\right)\right) \tag{2.9}$$

$$\rightarrow v_1, v_2, \dots, v_L: maximise\left(min\left(Sin^2\theta_{ij}, i \neq j\right)\right) \tag{2.10}$$

Both $|v_i - v_j|$ and $|v_i + v_j|$ are considered for minimum distance because the symbols that are used for transmission are generally QAM symbols and QAM symbols are positive and negative equi-probably.

The Grassmannian space $G(m, n)$ is the set of all n-dimensional subspaces of real m-dimensional Euclidean space $\mathbb{R}^m$. The optimisation problem in Equation (2.10) has been studied in detail by mathematicians under the technique called Grassmannian Line Packing [4].

The Grassmannian Line Packing of L lines gives L lines passing through the origin in the Grassmannian space $G(m,1)$ such that the minimum distance between any pair of lines is maximised. Here, the minimum distance between two lines is defined as the sine of the smaller of the two angles between the two lines. Taking the vectors $v_1, v_2, \dots, v_L$ along the L lines, the condition in Equation (2.10) is satisfied. Hence the solution for Grassmannian Line Packing is used to design the precoders for QAM symbols at each T_i.

2.2 Decoding at the Receiver

Consider the receiver R_1. Let x_i be the modulated symbol transmitted from T_i. Let n_1 be the noise added at R_1. Let the interference power of the remaining signal received from each T_i be $\alpha_i, i \neq 1$. Hence the received signal y_1 at R_1 is given by:

$$y_1^{K\times 1} = x_1 v_1 + \sum_{j=2}^{L} \sqrt{\alpha_j}(x_j v_j) + n_1 \tag{2.11}$$

It is assumed that the receivers have information about all the precoders. Since the precoder used by each T_i is known and the optimisation is done over $|v_i \cdot v_j|$, the initial choice for decoding is using Q^T.

$$Q^T * Q = \begin{bmatrix} v_1 \cdot v_1 & v_2 \cdot v_1 & \cdots & v_L \cdot v_1 \\ v_1 \cdot v_2 & v_2 \cdot v_2 & \cdots & v_L \cdot v_2 \\ \vdots & \vdots & \ddots & \vdots \\ v_1 \cdot v_L & v_2 \cdot v_L & \cdots & v_L \cdot v_L \end{bmatrix} \tag{2.12}$$

Normalising the power used for each symbol to 1, i.e. taking $|v_i \cdot v_i| = 1$,

$$Q^T * Q = \begin{bmatrix} 1 & Cos\theta_{21} & \cdots & Cos\theta_{L1} \\ Cos\theta_{12} & 1 & \cdots & Cos\theta_{L2} \\ \vdots & \vdots & \ddots & \vdots \\ Cos\theta_{1L} & Cos\theta_{2L} & \cdots & 1 \end{bmatrix} \tag{2.13}$$

The component of the received signal in the direction of v_1 is estimated to decode the symbol transmitted by T_1. Hence y_1 is multiplied by v_1^T to decode x_1. In general, the received signal at any T_i is multiplied by Q^T and the i^{th} value in the resulting matrix is decoded to the nearest symbol of the transmitted constellation.

The noise variance N_0 and total interference power at R_i, IP_i are defined as:

$$N_0 = 2\sigma^2 \tag{2.14}$$

$$IP_i = \sum_{j \neq i} \alpha_j \tag{2.15}$$

where σ^2 denotes the variance of real and imaginary part of noise separately in each dimension.

The bit error rates for some K and L values for $N_0 = 0.4$ (AWGN) and total interference power IP_1 = 0.4 at R_1(each α_i is taken to be equal) for QPSK are given in the following tables. The value of N_0 and IP_1 are chosen such that the BERs fall in comparable range to each other with enough variation to observe the general trend.

Table 2.1: BER for $\frac{K}{L} = \frac{2}{3}$

L=3	L=6	L=9	L=12	L=15
$\mathbf{2.2 \times 10^{-2}}$	3×10^{-3}	1.8×10^{-3}	9.3×10^{-4}	6×10^{-4}

Table 2.2: BER for $\frac{K}{L} = \frac{4}{5}$

L=5	L=10	L=15	L=20
$\mathbf{2.6 \times 10^{-3}}$	9×10^{-4}	6×10^{-4}	2.3×10^{-4}

These results are for R_1, but similar BER values are expected at other $R_i s$ due to the symmetry of the precoder matrices. For a fixed number of resource blocks K = 8, as number of users decreases from 12 to 10, the BER decreases. This trend follows for other noise and interference values as well. So, as expected, the error rates decreases as the number of users approaches the number of resource blocks, i.e. as K → L. For a fixed $\frac{K}{L}$, it can be observed that error rates decrease as L value increases. This general trend is also observed for other N_0 and IP. Theoretically, keeping the ratio of number of users to resource blocks fixed, increasing L will give better performance. Hence, effectively L symbols can be transmitted over K resource blocks more efficiently as L increases. But in practical scenarios, there are not many interferers. Hence, there is limitation on L and BER cannot be decreased beyond certain extent.

2.3 Zero-forcing Receiver

A zero-forcing receiver uses $Q^T * (Q * Q^T)^{-1}$ as the decoding matrix. The BER results for $Q^{2\times3}$ are inconclusive to decide between the two decoding methods.

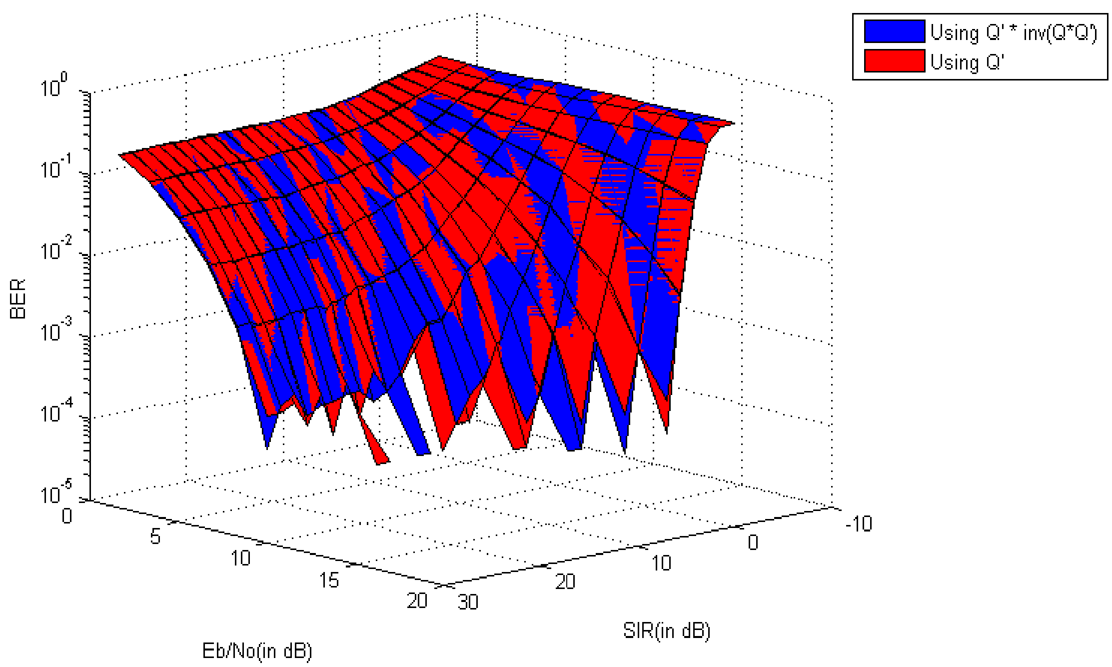


Figure 2.2: BER comparison between decoders Q^T and $Q^T * (Q * Q^T)^{-1}$ for $Q^{2\times3}$

So, $Q^{4\times6}$ is used for comparing the results of the two decoding methods. It can be seen from the graph below that the BER using $Q^T * (Q * Q^T)^{-1}$ for decoding is lower than BER using Q^T for decoding. This trend continues for higher order matrices also. Though the decoding is not specific to AWGN channels, it is taken that noise is Gaussian in the simulation.

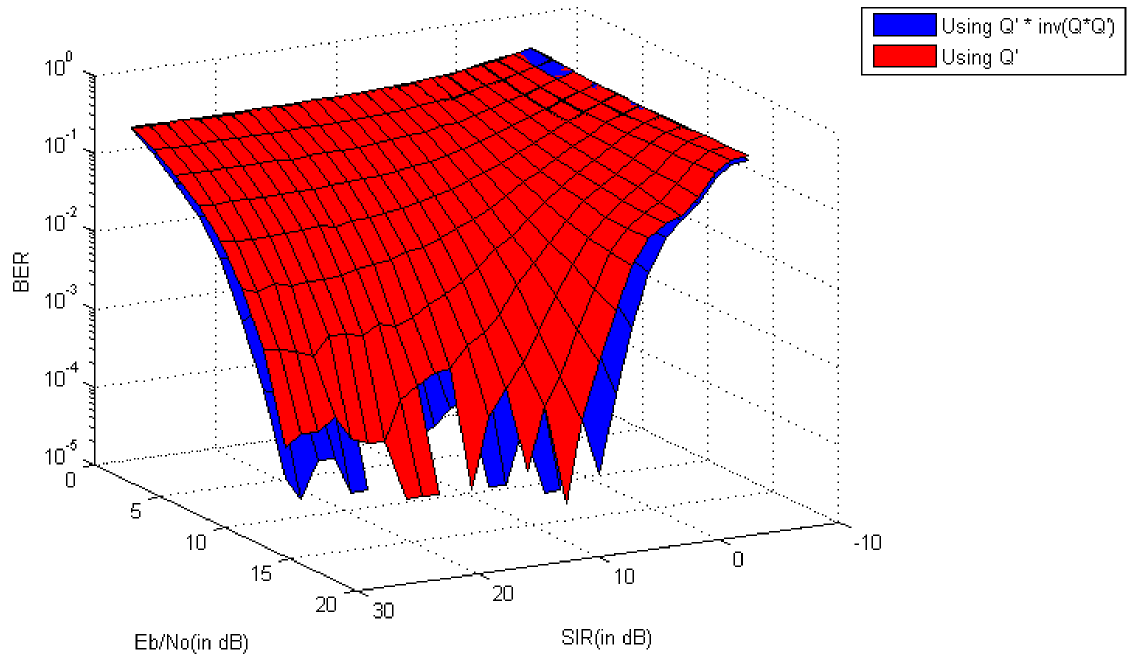


Figure 2.3: BER comparison between decoders Q^T and $Q^T * (Q * Q^T)^{-1}$ for $Q^{4\times6}$

The discrepancy in the case of $Q^{2\times3}$ is because:

$$Q^{2\times3} = \begin{bmatrix} 1 & 0.5 & -0.5 \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \tag{2.16}$$

$$\therefore Q^T * (Q * Q^T)^{-1} = \begin{bmatrix} \frac{2}{3} & 0 \\ -\frac{1}{3} & \frac{1}{\sqrt{3}} \\ -\frac{1}{3} & -\frac{1}{\sqrt{3}} \end{bmatrix} \tag{2.17}$$

$$\rightarrow Q^T * (Q * Q^T)^{-1} = \frac{2}{3} * Q^T \tag{2.18}$$

Since both the decoder matrices have elements in the same ratio, the BER for 2×3 case is same for both the cases. From now onwards, the zero-forcing receiver is preferentially used over the other decoder for decoding at the receiver.

2.4 3 User Interference

The general L user case has been studied till now. But in real life scenario, generally a 3 user case is encountered. Hence, the model is modified for a 3 user case. The combined precoder matrix Q is split into 3 parts, q_1, q_2 and q_3, each of size $K \times N_1$, $K \times N_2$ and $K \times N_3$ respectively. For symmetry purpose, it is assumed that $N_1 = N_2 = N_3 = \frac{L}{3} = N$. So, each user has a $K \times N$ precoder. $N = 1, K = 2$ is the simplest case. The combined Q is given by:

$$Q^{2\times3} = \begin{bmatrix} 1 & 0.5 & -0.5 \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \tag{2.19}$$

$$q_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad q_2 = \begin{bmatrix} 0.5 \\ \frac{\sqrt{3}}{2} \end{bmatrix}, \qquad q_3 = \begin{bmatrix} -0.5 \\ \frac{\sqrt{3}}{2} \end{bmatrix} \tag{2.20}$$

Maintaining the ratio of $\frac{N}{K} = \frac{1}{2}$, the next possible precoder is $Q^{4\times6}$. It is given by:

$$Q^{4\times6} = \begin{bmatrix} -0.4475 & 0.0031 & 0.0776 & -0.6915 & 0.5282 & -0.6108 \\ -0.0361 & 0.8963 & -0.4002 & -0.0415 & 0.5322 & 0.4546 \\ 0.7259 & 0.4312 & -0.1584 & -0.5780 & -0.4531 & -0.3052 \\ 0.5211 & 0.1039 & 0.8993 & -0.4313 & 0.4821 & 0.5719 \end{bmatrix} \tag{2.21}$$

$$q_1 = \begin{bmatrix} -0.4475 & 0.0031 \\ -0.0361 & 0.8963 \\ 0.7259 & 0.4312 \\ 0.5211 & 0.1039 \end{bmatrix}, \qquad q_2 = \begin{bmatrix} 0.0776 & -0.6915 \\ -0.4002 & -0.0415 \\ -0.1584 & -0.5780 \\ 0.8993 & -0.4313 \end{bmatrix},$$
$$q_3 = \begin{bmatrix} 0.5282 & -0.6108 \\ 0.5322 & 0.4546 \\ -0.4531 & -0.3052 \\ 0.4821 & 0.5719 \end{bmatrix} \tag{2.22}$$

Here, each user sends 2 symbols using its own 4×2 precoder matrix. The MMSE receiver matrix $Q^T * (Q * Q^T + \sigma^2 * I_K)^{-1}$ is used for decoding when the channel is assumed to add Gaussian noise having variance σ^2.

The BERs for $Q^{2\times3}$and $Q^{4\times6}$ for QPSK are compared below:

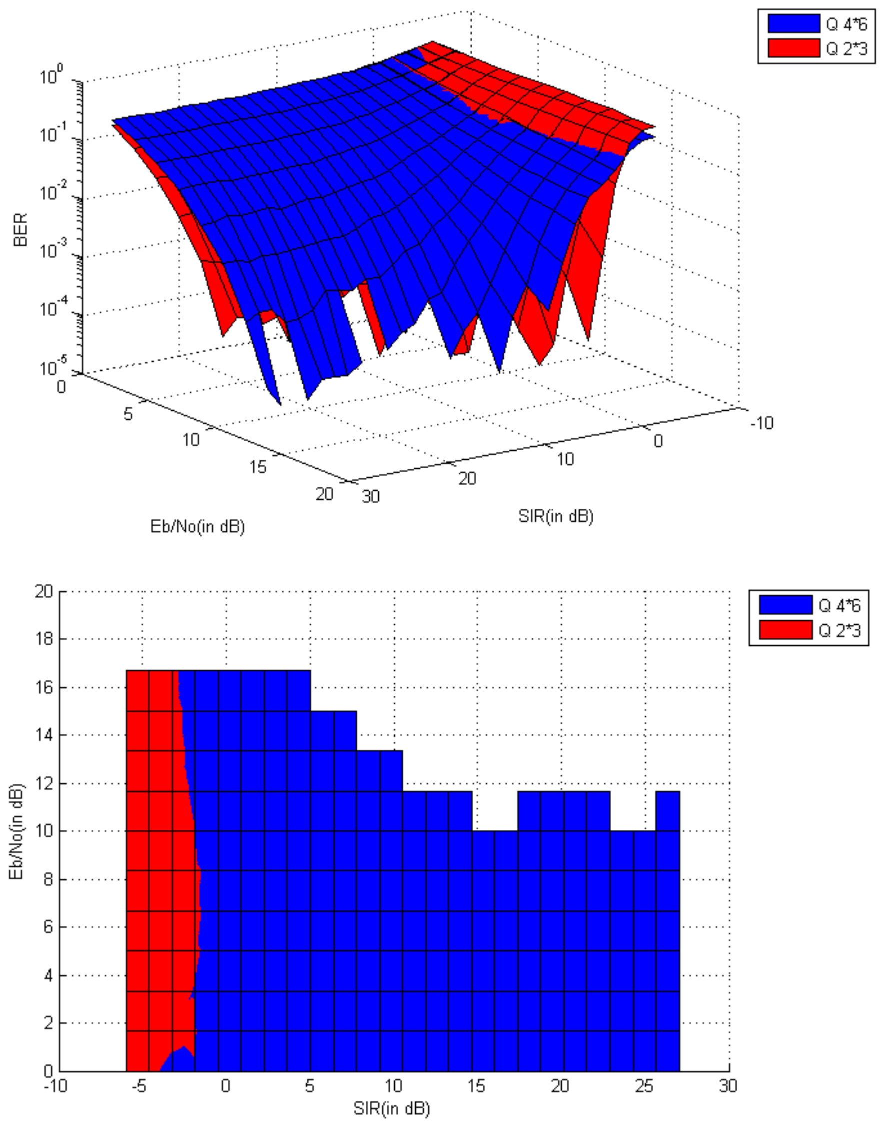


Figure 2.4: BER comparison between $Q^{4\times6}$ and $Q^{2\times3}$

(a) 3-D view (b)Top view

From the top view, it is clear that above 0dB SIR, $Q^{4\times6}$ has higher BER when compared to $Q^{2\times3}$. The same trend continues for higher N values also.

2.5 Joint decoding

Both the decoding methods discussed above are symbol by symbol detection. For $N = 1, K = 2$ case, joint detection is not possible because every transmitter transmits only 1 symbol. But, since there is more than 1 symbol transmitted by one transmitter in $N = 2, K = 4$ case, joint detection of the symbols can be done. Let x_1, x_2 be the symbols transmitted by T_1. Let x_3, x_4 and x_5, x_6 be the symbols transmitted by T_2 and T_3. Consider the receiver R_1. Let y be the received signal at R_1 and $n^{4\times 1}$ be the AWGN added.

$$y = q_1 * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \sqrt{\alpha} * q_2 * \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \sqrt{\beta} * q_3 * \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} + n \tag{2.23}$$

Joint decoding is explained for the above case for a QPSK constellation. The joint decoding is similar for other constellations also. Since it is assumed that interference information is not available at the receiver, the whole interference term is considered as noise. Suppose $L^{2\times 16}$ be the matrix containing the set of all possible combination of 2 QPSK symbols. l_i is defined as the i_{th} column of the matrix L . d_i is defined as

$$d_i = \| q_1^T * (QQ^T + \sigma^2 I_N)^{-1} * (y - q_1 * l_i) \| \tag{2.24}$$

The received signal is decoded to the l_i corresponding to the smallest d_i.

The performance of these decoding techniques is further discussed in Chapter 4.

CHAPTER 3

REDUCED LOG LIKELIHOOD RATIOS

Consider the case of a single transmitter and receiver. The data is modulated by the transmitter and sent through the channel. Let $\boldsymbol{S}$ denote the set of symbols in the constellation used by the transmitter. The receiver receives the sent signal added with noise. It is assumed that the noise is Gaussian for the study.

$$y = x + n, x \in \boldsymbol{S} \tag{3.1}$$

The pdf of noise, n is given by

$$f_n(u) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{u^2}{2\sigma^2}}, \qquad u \in \mathbb{R} \tag{3.2}$$

for real case and

$$f_n(u) = \frac{1}{\pi\sigma^2} e^{-\frac{|u|^2}{\sigma^2}}, \qquad u \in \mathbb{C} \tag{3.3}$$

for complex case. Here σ^2 is the variance for the noise pdf.

3.1 LLR for General Case

Log likelihood Ratio (LLR) is defined as the ratio of probability of the sent signal being one of the symbols to the probability of it being any other symbol. Let $\boldsymbol{S} = \{s_0, s_1, \dots, s_K\}$ be the set of all the constellation symbols used by transmitter.

$$LLR(y^i) = log\left(\frac{P(x = s_i|y)}{P(x = s_j|y, j \neq i)}\right) \tag{3.4}$$

Generally the symbols are further decoded into bits. Suppose the received symbol is decoded into a binary string $z_m z_{m-1} \dots z_0$. To find the LLR of the i^{th}bit in the decoded string, $\boldsymbol{S}$ is partitioned into two sets such that, $S_i^1 \subset \boldsymbol{S}$ contains constellation points with 1 in the i^{th} position and $S_i^0 \subset \boldsymbol{S}$ contains constellation points with 0 in the i^{th} position. The log likelihood ratio (LLR) of the i^{th} bit is defined as:

$$LLR(z_i) = \log\left(\frac{\sum_{s_i \in S_i^1} P(x = s_i|y)}{\sum_{s_i \in S_i^0} P(x = s_i|y)}\right) \tag{3.5}$$

Calculating $P(x = s_i|y)$ is difficult. So, this probability is simplified as:

$$P(x = s_i|y) = \frac{P(y|x = s_i) \times P(x = s_i)}{P(y)} \tag{3.6}$$

Every symbol in the constellation is assumed to be used with equal probability. Hence, combining equation (3.5) and equation (3.6), $LLR(z_i)$ can be reduced to:

$$LLR(z_i) = log\left(\frac{\sum_{s_i \in S_i^1} P(y|x = s_i)}{\sum_{s_i \in S_i^0} P(y|x = s_i)}\right) \tag{3.7}$$

3.2 Reduced LLR (RLLR) for BPSK

Consider the case where the constellation is BPSK. Let the noise variance σ^2 be taken as N_0. The encoded constellation will look as below:

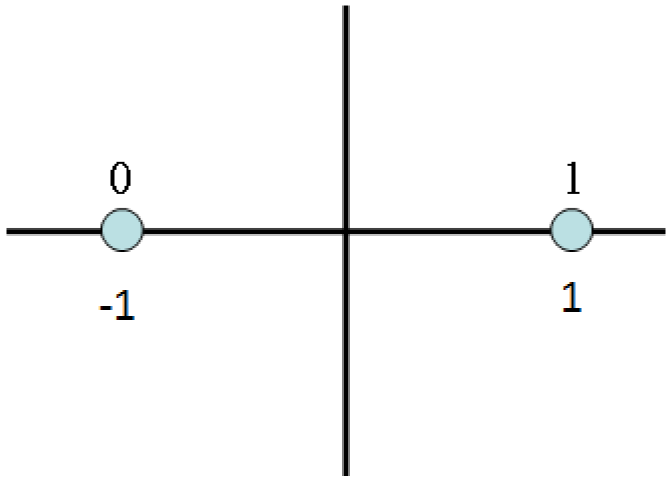


Figure 3.1: BPSK encoded in bits

Using equation (3.7), the LLR expression for decoded bit z_0 is given by:

$$LLR(z_0) = log\left(\frac{P(y|x=1)}{P(y|x=-1)}\right) \tag{3.8}$$

$$\rightarrow LLR(z_0) = log\left(\frac{f_n(y-1)}{f_n(y+1)}\right) \tag{3.9}$$

$$\rightarrow LLR(z_0) = log\left(\frac{e^{-\frac{(y-1)^2}{2\sigma^2}}}{e^{-\frac{(y+1)^2}{2\sigma^2}}}\right) \tag{3.10}$$

$$\rightarrow RLLR(z_0) = LLR(z_0) = log\left(e^{\frac{2y}{\sigma^2}}\right) \tag{3.11}$$

Decision is made on z_0 depending on whether $RLLR(z_0) > 0$ or $RLLR(z_0) < 0$. $z_0 = 1$ and $z_0 = 0$ respectively. The results of LLR decision making is as shown below:

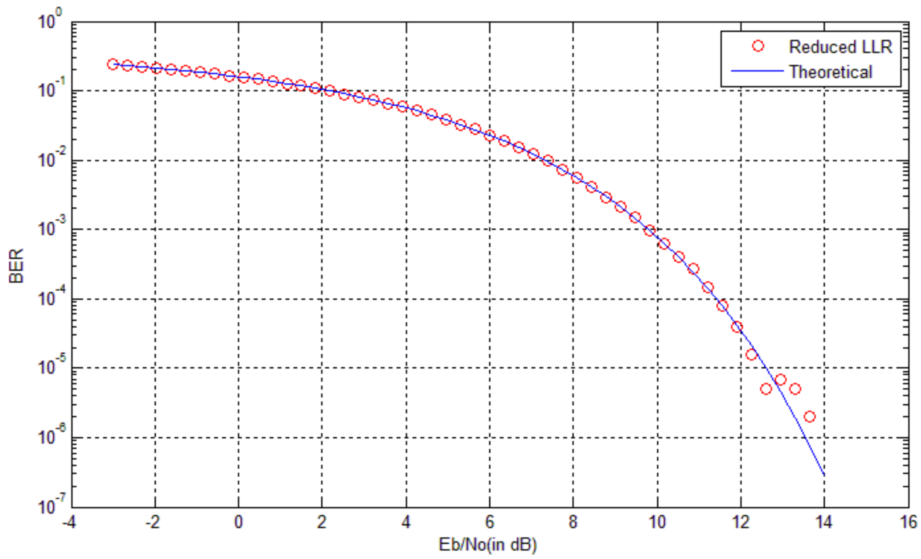


Figure 3.2: BER v/s $\frac{E_b}{N_0}$ for BPSK using RLLR decoding

3.3 Reduced-LLR (RLLR) for QPSK

When the constellation used by the transmitter becomes complex, the noise splits into real and imaginary parts with total variance of N_0. Hence the noise variance in each part is $\sigma^2 = \frac{N_0}{2}$. Here the bits are Gray-coded so that an error in detection creates an error in minimum number of bits. The encoded QPSK constellation looks as below:

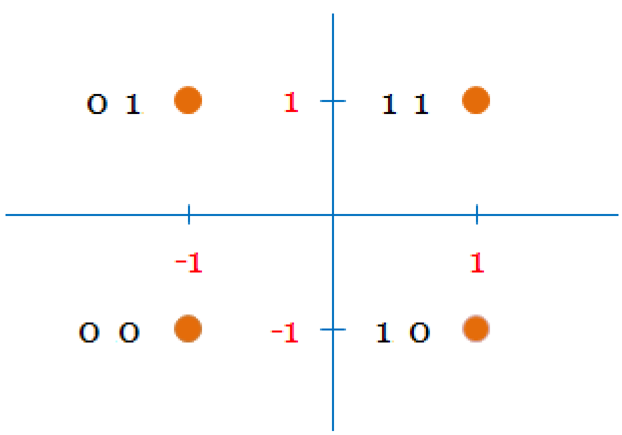


Figure 3.3: QPSK with Gray-coding

Let the received signal be $y = a + ib$. Let the decoded bit string be $z_1 z_0$. The LLR expressions are given by:

$$LLR(z_0) = log\left(\frac{P(y|x = 11) + P(y|x = 01)}{P(y|x = 10) + P(y|x = 00)}\right) \quad (3.12)$$

$$P(y|_{x=11}) = P(n = (a - 1) + i(b - 1)) \quad (3.13)$$

$$\rightarrow P(y|_{x=11}) = \frac{1}{\pi\sigma^2} e^{-\frac{(a-1)^2+(b-1)^2}{\sigma^2}} \quad (3.14)$$

$$\rightarrow P(y|_{x=11}) = \frac{1}{\sqrt{\pi\sigma^2}} e^{-\frac{(a-1)^2}{\sigma^2}} * \frac{1}{\sqrt{\pi\sigma^2}} e^{-\frac{(b-1)^2}{\sigma^2}} \quad (3.15)$$

Let us define a new function $g(x, y)$ to simplify the expressions for LLR.

$$g(x, y) = e^{-\frac{(x-y)^2}{\sigma^2}} \quad (3.16)$$

Using Equation (3.14) and Equation (3.15), Equation (3.12) can be re-written as:

$$LLR(z_0) = log\left(\frac{g(a, 1) * g(b, 1) + g(a, -1) * g(b, 1)}{g(a, 1) * g(b, -1) + g(a, -1) * g(b, -1)}\right) \quad (3.17)$$

$$\rightarrow LLR(z_0) = log\left(\frac{\big(g(a, 1) + g(a, -1)\big) * g(b, 1)}{\big(g(a, 1) + g(a, -1)\big) * g(b, -1)}\right) \quad (3.18)$$

$$\rightarrow RLLR(z_0) = LLR(z_0) = log\left(\frac{g(b, 1)}{g(b, -1)}\right) \quad (3.19)$$

The reduced LLR (RLLR) for QPSK using Gray-coding simplifies to 1-term by 1-term form, thus reducing the complexity of calculating LLR.

In a similar way, RLLR for z_1 can be calculated to be:

$$RLLR(z_1) = log\left(\frac{g(a,1)}{g(a,-1)}\right) \quad (3.20)$$

Energy per bit (E_b) for QPSK as shown above is $\frac{1^2+1^2}{2} = 1$. The theoretical value of BER for QPSK [7] is given by:

$$BER_{th} = \frac{1}{2} erfc\left(\frac{1}{\sqrt{2}\sigma}\right), \quad (3.21)$$

$$\text{where } erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt \quad (3.22)$$

The BER of Reduced LLR decoding coincides with the minimum distance decoding as well as the theoretical estimate of BER, validating the expression obtained for $RLLR$.

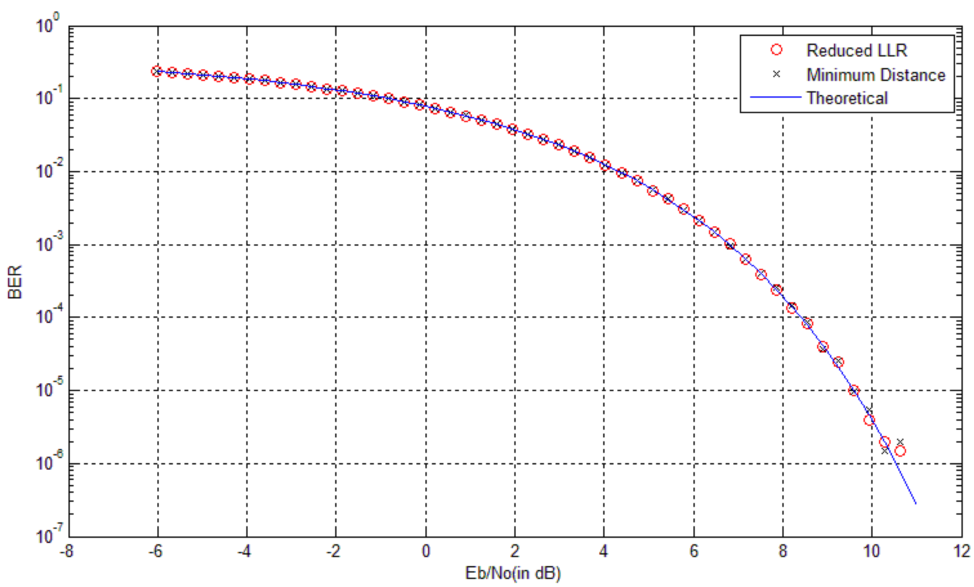


Figure 3.4: BER v/s $\frac{E_b}{N_0}$ for QPSK using RLLR decoding

3.4 Reduced-LLR (RLLR) for 16-QAM

Gray-coded 16-QAM is transmitted and let the received signal be $y = a + ib$.

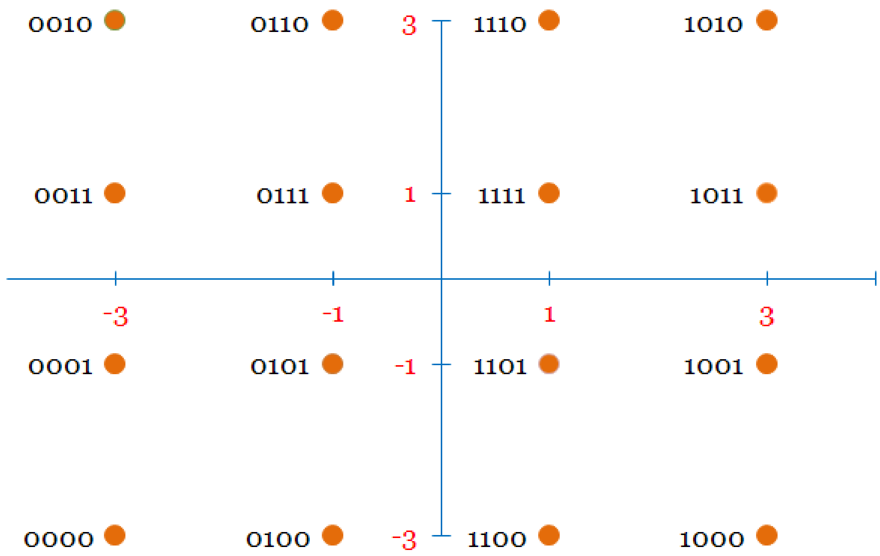


Figure 3.5: 16-QAM with Gray-coding

4 bits are needed for transmitting a 16-QAM symbol. So, let the decoded bit string be $z_3z_2z_1z_0$. The LLR expression for z_0 is given by:

$$LLR(z_0) = log\left(\frac{NR_1 + NR_2}{DR_1 + DR_2}\right), \tag{3.23}$$

$$where\ NR_1 = P(y|x = 0001) + P(y|x = 0101) + P(y|x = 0001) + P(y|x = 0101), \tag{3.24}$$

$$NR_2 = P(y|x = 0011) + P(y|x = 0111) + P(y|x = 0011) + P(y|x = 0111), \tag{3.25}$$

$$DR_1 = P(y|x = 0000) + P(y|x = 0100) + P(y|x = 0000) + P(y|x = 0100), \tag{3.26}$$

$$DR_2 = P(y|x = 0010) + P(y|x = 0110) + P(y|x = 0010) + P(y|x = 0110) \tag{3.27}$$

$$NR_1 = g(a,-3)g(b,-1) + g(a,-1)g(b,-1) + g(a,1)g(b,-1) + g(a,3)g(b,-1) \tag{3.28}$$

$$\rightarrow NR_1 = g(b,-1) * \left(g(a,-3) + g(a,-1) + g(a,1) + g(a,3)\right) \tag{3.29}$$

It can be observed that NR_2 also has $g(a,-3) + g(a,-1) + g(a,1) + g(a,3)$ as a factor. Also, $DR_1\ and\ DR_2$ have the above mentioned factor. Hence, the reduced LLR expression is:

$$RLLR(z_0) = log\left(\frac{g(b,-1) + g(b,1)}{g(b,-3) + g(b,3)}\right) \tag{3.30}$$

Using a similar approach, RLLR of $z_1$is calculated to be:

$$RLLR(z_1) = log\left(\frac{g(b,3) + g(b,1)}{g(b,-3) + g(b,-1)}\right) \tag{3.31}$$

For calculating the RLLR of z_3 and z_4, $g(b,-3) + g(b,-1) + g(b,1) + g(b,3)$ is taken as the common factor in both numerator and denominator. The reduced LLR expressions for z_2 and z_3 are:

$$RLLR(z_2) = log\left(\frac{g(a,-1) + g(a,1)}{g(a,-3) + g(a,3)}\right) \tag{3.32}$$

$$RLLR(z_3) = log\left(\frac{g(a,-1) + g(a,1)}{g(a,-3) + g(a,3)}\right) \tag{3.33}$$

The reduced LLR (RLLR) for 16-QAM using Gray-coding simplifies to a 2-terms by 2-terms form, thus reducing the complexity of calculating LLR from calculating an 8-terms by 8-terms expression to a 2-terms by 2-terms expression.

Average energy per bit (E_b) for 16-QAM is $\frac{1^2+3^2}{4} = \frac{5}{2}$. The theoretical value of BER for 16-QAM [7] is given by:

$$BER_{th} = \frac{3}{8} erfc\left(\frac{1}{\sqrt{2}\sigma}\right), \tag{3.34}$$

$$\text{where } erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt \tag{3.35}$$

The BER of Reduced LLR decoding coincides with the minimum distance decoding as well as the theoretical estimate of BER, validating the expression obtained for $RLLR$.

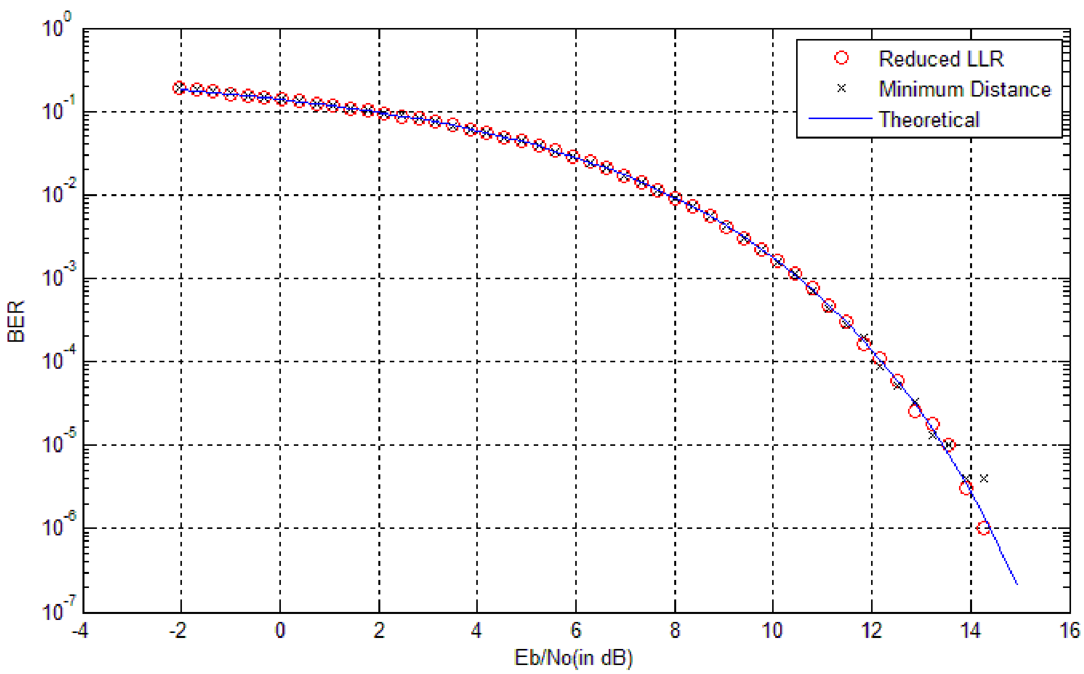


Figure 3.6: BER v/s $\frac{E_b}{N_0}$ for 16-QAM using RLLR decoding

3.5 Reduced-LLR (RLLR) for 64-QAM

Gray-coded 64-QAM is transmitted and let the received signal be $y = a + ib$.

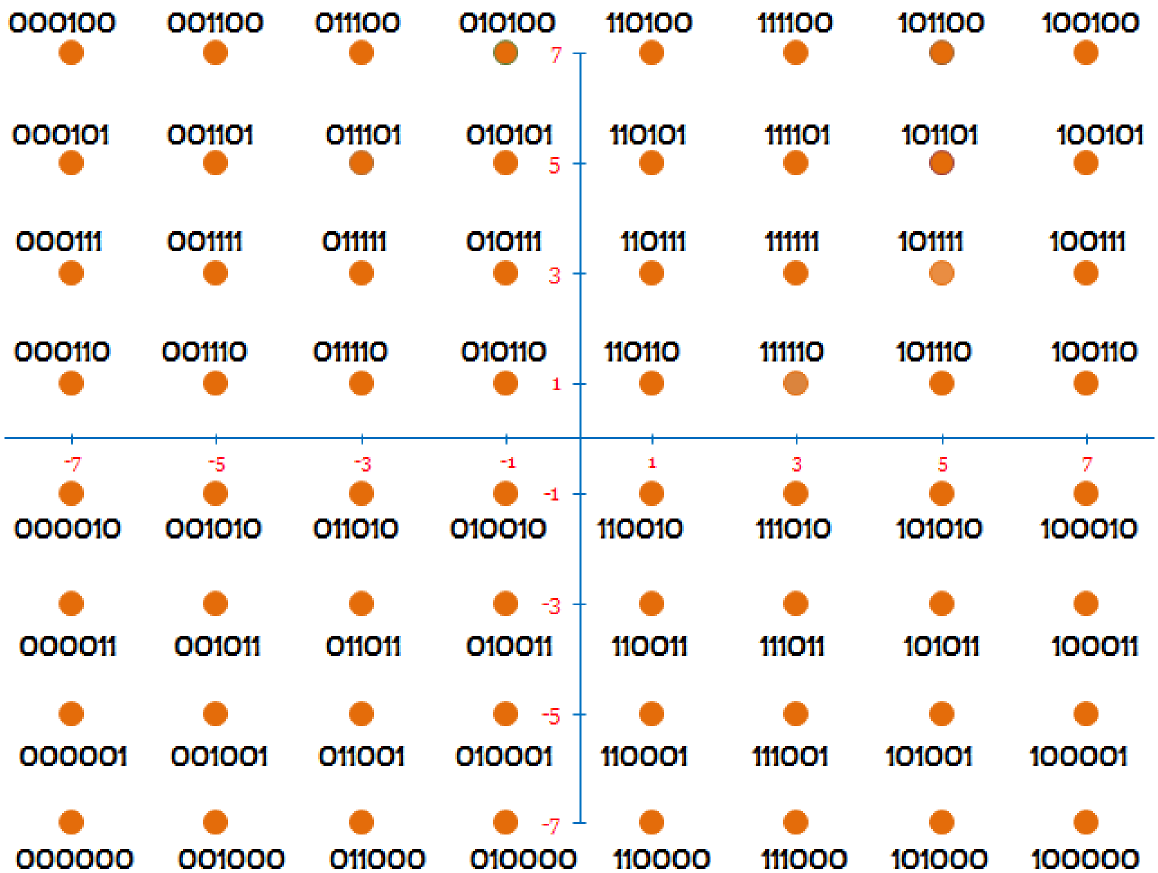


Figure 3.7: 64-QAM with Gray-coding

6 bits are needed for transmitting a 64-QAM symbol. So, let the decoded bit string be $z_5\, z_4\, z_3\, z_2\, z_1\, z_0$. The LLR expression for z_0 is given by:

$$LLR(z_0) = log\left(\frac{NR_1 + NR_2 + NR_3 + NR_4}{DR_1 + DR_2 + DR_3 + DR_4}\right), \tag{3.36}$$

$where\ NR_1 = \sum_{t=\{*001\}} P(y|x = t)$, where $*$ denotes the set of all the 8 3-bit combinations. (3.37)

$$NR_2 = \sum_{t=\{*011\}} P(y|x=t)\,, NR_3 = \sum_{t=\{*111\}} P(y|x=t)\ \ and$$

$$NR_4 = \sum_{t=\{*101\}} P(y|x=t) \tag{3.38}$$

$$DR_1 = \sum_{t=\{*000\}} P(y|x=t)\,, DR_2 = \sum_{t=\{*010\}} P(y|x=t)\,,$$

$$DR_3 = \sum_{t=\{*110\}} P(y|x=t)\,, \tag{3.39}$$

$$NR_4 = \sum_{t=\{*100\}} P(y|x=t)$$

In all the above expressions, $*$ denotes the set of all the 8 3-bit combinations.

$$\begin{aligned} NR_1 = {} & g(a,-7)g(b,-5) + g(a,-5)g(b,-5)g(a,-3)g(b,-5) \\ & + g(a,-1)g(b,-5) + g(a,1)g(b,-5) + g(a,3)g(b,-5) \\ & + g(a,5)g(b,-5) + g(a,7)g(b,-5) \end{aligned} \tag{3.40}$$

$$\begin{aligned} \rightarrow NR_1 = {} & g(b,-5) \\ & * (g(a,-7) + g(a,-5) + g(a,-3) + g(a,-1) + g(a,1) \\ & + g(a,3) + g(a,5) + g(a,7)) \end{aligned} \tag{3.41}$$

Also, $NR_2, NR_3, NR_4, DR_1, DR_2, DR_3$ and DR_4 have the factor $g(a,-7) + g(a,-5) + g(a,-3) + g(a,-1) + g(a,1) + g(a,3) + g(a,5) + g(a,7)$.

Hence, the reduced LLR expression is:

$$RLLR(z_0) = log\left(\frac{g(b,-5) + g(b,-3) + g(b,3) + g(b,5)}{g(b,-7) + g(b,-1) + g(b,1) + g(b,7)}\right) \quad (3.42)$$

Using a similar approach, and cancelling out the common term $g(a,-7) + g(a,-5) + g(a,-3) + g(a,-1) + g(a,1) + g(a,3) + g(a,5) + g(a,7)$, RLLR of z_1 and z_2 can be calculated to be:

$$RLLR(z_1) = log\left(\frac{g(b,-3) + g(b,-1) + g(b,1) + g(b,3)}{g(b,-7) + g(b,-5) + g(b,5) + g(b,7)}\right) \quad (3.43)$$

$$RLLR(z_2) = log\left(\frac{g(b,1) + g(b,3) + g(b,5) + g(b,7)}{g(b,-1) + g(b,-3) + g(b,-5) + g(b,-7)}\right) \quad (3.44)$$

For calculating the RLLR of z_3, z_4 and z_5, $g(b,-7) + g(b,-5) + g(b,-3) + g(b,-1) + g(b,1) + g(b,3) + g(b,5) + g(b,7)$ is taken as the common factor in both numerator and denominator. The reduced LLR expressions for z_3, z_4 and z_5 are:

$$RLLR(z_3) = log\left(\frac{g(a,-5) + g(a,-3) + g(a,3) + g(a,5)}{g(a,-7) + g(a,-1) + g(a,1) + g(a,7)}\right) \quad (3.45)$$

$$RLLR(z_4) = log\left(\frac{g(a,-3) + g(a,-1) + g(a,1) + g(a,3)}{g(a,-7) + g(a,-5) + g(a,5) + g(a,7)}\right) \quad (3.46)$$

$$RLLR(z_5) = log\left(\frac{g(a,1) + g(a,3) + g(a,5) + g(a,7)}{g(a,-1) + g(a,-3) + g(a,-5) + g(a,-7)}\right) \quad (3.47)$$

The reduced LLR (RLLR) for 64-QAM using Gray-coding simplifies to a 4-terms by 4-terms form, thus reducing the complexity of calculating LLR from calculating a 32-terms by 32-terms expression to a 4-terms by 4-terms expression.

Average energy per bit (E_b) for 64-QAM is $\frac{1^2+3^2+5^2+7^2}{2\times 6} = 7$. The theoretical value of BER [7] for 64-QAM is given by:

$$BER_{th} = \frac{15}{64} erfc\left(\frac{1}{\sqrt{2}\sigma}\right), \quad (3.48)$$

$$\text{where } erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt \quad (3.49)$$

The BER of Reduced LLR decoding agrees with the minimum distance decoding as well as the theoretical estimate of BER, validating the expression obtained for $RLLR$.

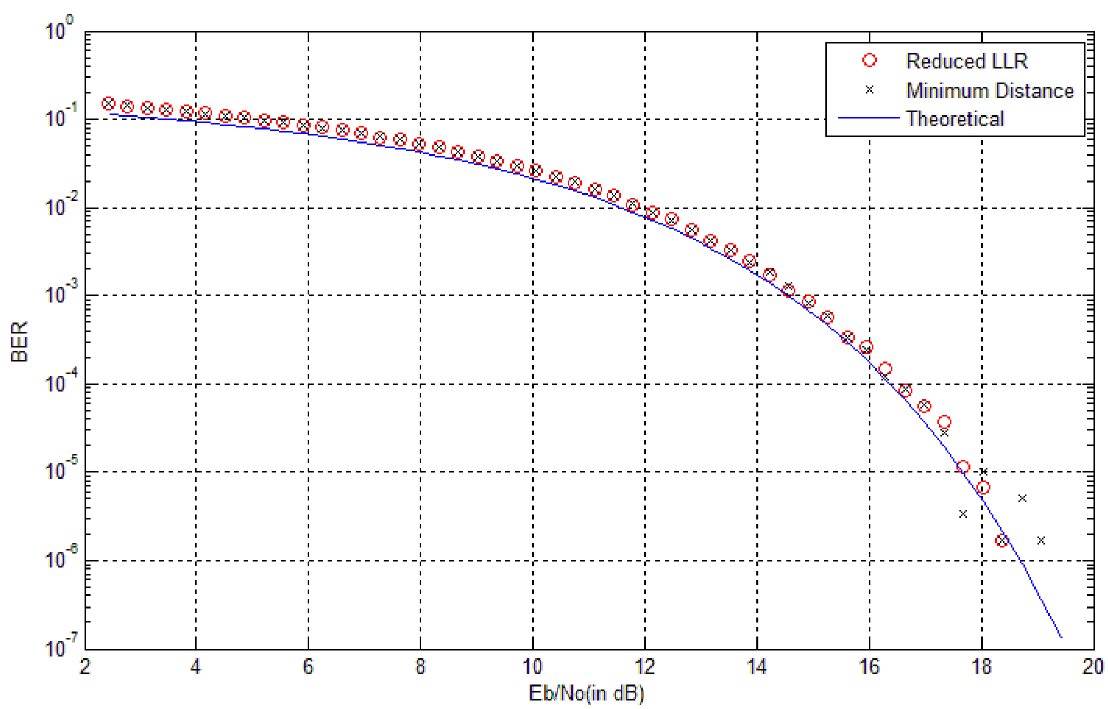


Figure 3.8: BER v/s $\frac{E_b}{N_0}$ for 64-QAM using RLLR decoding

CHAPTER 4

RESULTS AND CONCLUSION

The BER using $Q^{2\times3}$ as compared to the BER obtained by using ICBM for QPSK is:

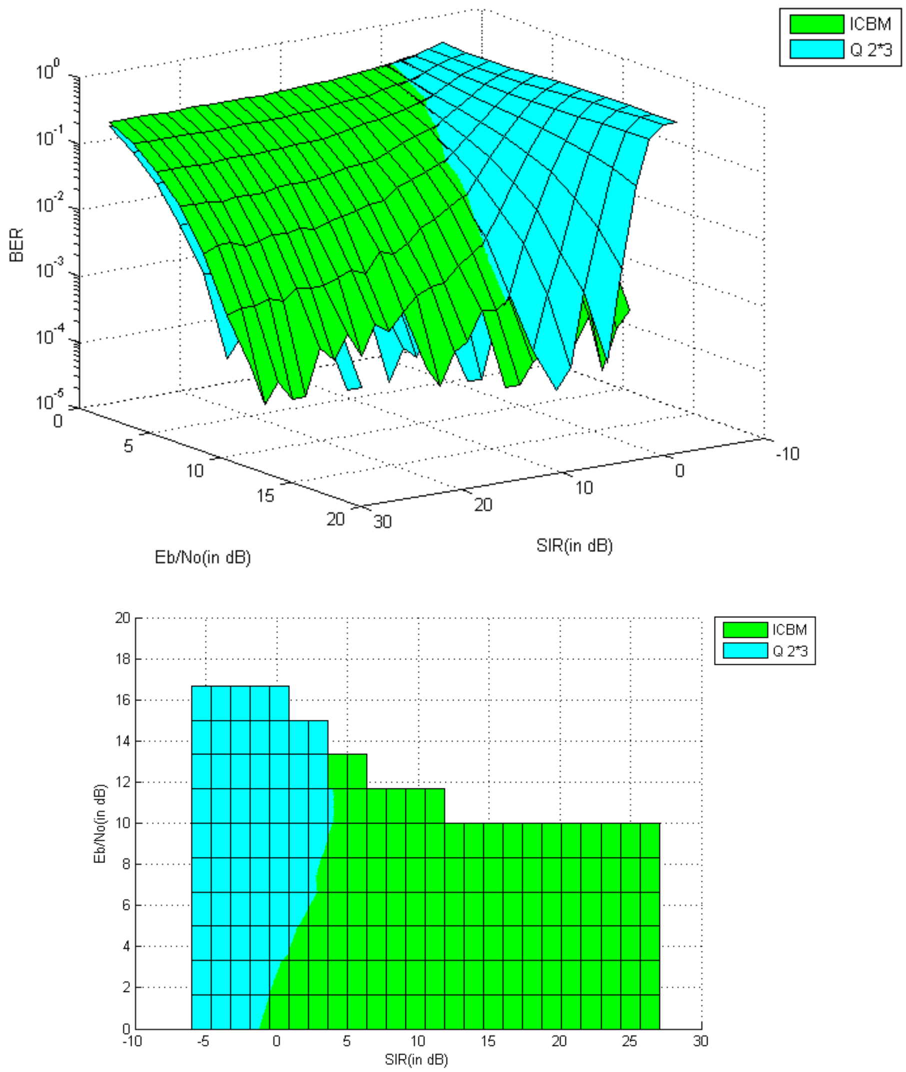


Figure 4.1: BER comparison between ICBM and $Q^{2\times3}$

(a) 3-D view (b) Top view

The results show that $Q^{2\times3}$ performs better in low interference regions, but as SIR goes below 0dB, ICBM performs better. One of the reasons for this is the power cancelling effect seen in the case of $Q^{2\times3}$. Consider a situation where all the 3 transmitters are trying to send $1 + 1i$. Case 1: $\alpha = 0.1$, $\beta = 0.1$.

$$y = \begin{bmatrix} 1 & 0.5 & -0.5 \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} * \begin{bmatrix} 1 + 1i \\ \sqrt{0.1}(1 + 1i) \\ \sqrt{0.1}(1 + 1i) \end{bmatrix} + n \tag{4.1}$$

$$y = \begin{bmatrix} 0.684 + 0.684i \\ 0 \end{bmatrix} + n \tag{4.2}$$

The total power of the signal received is 0.935. Case 2: $\alpha = 0.9$, $\beta = 0.9$

$$y = \begin{bmatrix} 1 & 0.5 & -0.5 \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} * \begin{bmatrix} 1 + 1i \\ \sqrt{0.9}(1 + 1i) \\ \sqrt{0.9}(1 + 1i) \end{bmatrix} + n \tag{4.3}$$

$$y = \begin{bmatrix} 0.0513 + 0.0513i \\ 0 \end{bmatrix} + n \tag{4.4}$$

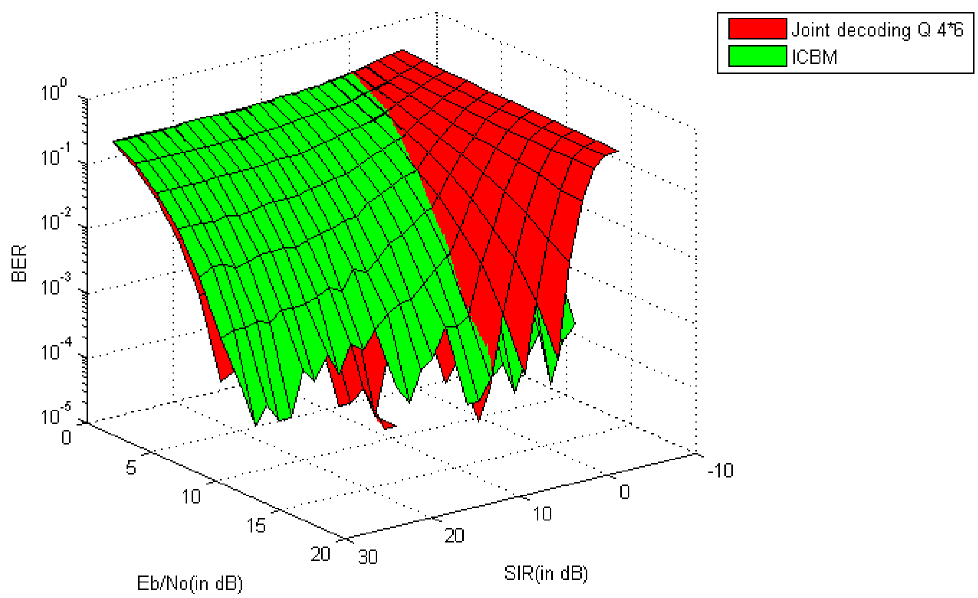
The total power of the received signal is 0.0053. But still, the signal will still be decoded correctly if the noise is low enough. Due to this effect $Q^{2\times3}$ becomes highly vulnerable to noise at higher interference.

ICBM matrix:

$$Q^{ICBM} = \begin{bmatrix} -1.0945 & 0.5479 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.0945 & 0.5479 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.0945 & 0.5479 \\ 0.3174 & 0.6319 & 0.3174 & 0.6319 & 0.3174 & 0.6319 \end{bmatrix} \tag{4.5}$$

Since, ICBM has at least one independent direction for every user, the energy of the signal does not go low near the receiver at least in one direction.

The BER of joint decoding using $Q^{4\times6}$ and ICBM is compared below:



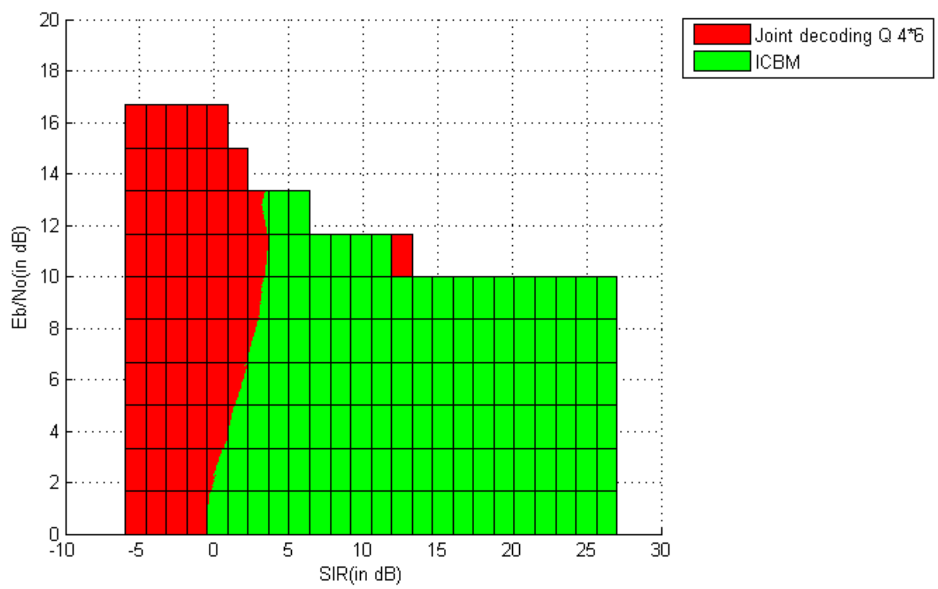


Figure 4.2: BER comparison between ICBM and joint decoding using $Q^{4\times6}$

(a) 3-D view (b) Top view

The results show that $Q^{4\times6}$ performs better in low interference regions, but as SIR goes below 0dB, ICBM performs better. The power cancelling effect seen in $Q^{2\times3}$ can also be seen in $Q^{4\times6}$ case. This is because of the symmetry of the chosen vectors. Joint decoding of higher dimension vectors increases the symbols to be jointly decoded, hence tremendously increasing the complexity of decoding. Also, the BER performance does not show much improvement. Hence, joint decoding is stopped at $Q^{4\times 6}$. For symbol by symbol detection, it has already been seen that BER increases as K increases for a 3 user case. So, $Q^{2\times 3}$ is the optimum vector for symbol by symbol detection.

APPENDIX A

Optimum Grassmannian Line Packing of 3 and 6 lines in $G(2,1)$ and $G(4,1)$ respectively are:

$$Q^{2\times 3} = \begin{bmatrix} 1 & 0.5 & -0.5 \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$Q^{4\times 6} = \begin{bmatrix} -0.4475 & 0.0031 & 0.0776 & -0.6915 & 0.5282 & -0.6108 \\ -0.0361 & 0.8963 & -0.4002 & -0.0415 & 0.5322 & 0.4546 \\ 0.7259 & 0.4312 & -0.1584 & -0.5780 & -0.4531 & -0.3052 \\ 0.5211 & 0.1039 & 0.8993 & -0.4313 & 0.4821 & 0.5719 \end{bmatrix}$$

APPENDIX B

Matlab funcitons:

In the following codes,

vec_in= Q

alph= α

bet= β

s2= σ^2

%Function to calculate BER by using $Q^T * (QQ^T + \sigma^2 * I_N)$ as
%decoding matrix for QPSK

```
function [e] = error_q(vec_in,alph,bet,s2)
    iter=20000;
    len=zeros(iter,1);
    siz=size(vec_in);
    n=siz(2);
    noi=zeros(siz(1),iter);
    for j=1:siz(1)
      noi(j,:)=sqrt(s2/2)*(randn(iter,1)+1i*randn(iter,1));
    end
    for j=1:iter
        r=rand_qpsk(n);
        r_int=zeros(n,1)';
        r_int=[r(1:n/3); sqrt(alph)*r(n/3+1:2*n/3);
          sqrt(bet)*r(2*n/3+1:n)];
        intermediate=vec_in*r_int;
```

```
        recd=intermediate+noi(:,j);
        rec1=vec_in'*(inv(vec_in*vec_in'+
          s2*eye(siz(1))))*recd;
        rec=rec1;
        rec(real(rec)<=0&imag(rec)<=0)=-1-1i;
        rec(real(rec)<=0&imag(rec)>=0)=-1+1i;
        rec(real(rec)>=0&imag(rec)<=0)=1-1i;
        rec(real(rec)>=0&imag(rec)>=0)=1+1i;

        len(j)=(n/3)-length(find(rec(1:n/3)==r(1:n/3)));
    end
    err = mean(len);
    e = err*3/n;
end



%Function to calculate BER for joint detection using Q
%for QPSK
function e = error_joint_detection(vec_in,alph,bet,s2)
    iter=20000;
    len=zeros(iter,1);
    siz=size(vec_in);
    n=siz(2);
    noi=zeros(siz(1),iter);

    r_ref1=zeros(4,1);
    r_ref1(1,1)=-1-1i;
    r_ref1(2,1)=-1+1i;
    r_ref1(3,1)=1-1i;
    r_ref1(4,1)=1+1i;
    g1=meshgrid(r_ref1);
    g2=transpose(g1);
```

```
    g1=reshape(g1,1,16);
    g2=reshape(g2,1,16);
    qpsk_2=[g1;g2];
    re_ref=zeros(n,16);
    re_ref(1:2,:)=qpsk_2;
    rec=zeros(n,1);

    for j=1:siz(1)
      noi(j,:)=sqrt(s2/2)*(randn(iter,1)+1i*randn(iter,1));
    end
    for j=1:iter
        r=rand_qpsk(n);
        r_int=zeros(n,1);
        r_int=[r(1:n/3); sqrt(alph)*r(n/3+1:2*n/3);
          sqrt(bet)*r(2*n/3+1:n)];

        intermediate=vec_in*r_int;
        recd=intermediate+noi(:,j);
        recd_rep=repmat(recd,1,16);
        rec1=vec_in'*(inv(vec_in*vec_in'+
          s2*eye(siz(1))))*(recd_rep-vec_in*re_ref);
        gen1=rec1(1:2,:);
        gen2=dot(gen1,gen1);
        in1=find(gen2==min(gen2));
        rec(1:n/3)=re_ref(1:2,in1(end));

        len(j)=(n/3)-length(find(rec(1:n/3)==r(1:n/3)));
    end
    err = mean(len);
    e = err*3/n;

end
```

REFERENCES

[1] Telecom Subscription Data as of May 31st, 2012" http://www.trai.gov.in/WriteReadData/PressRealease/Document/PR-TSD-May12.pdf

[2] D. Ring and W. Young, "Mobile telephony-wide area coverage," Bell Laboratories Technical Memorandum, Tech. Rep., 1947.

[3] Arun B Ayyar, "Techniques to Manage Co-Channel Interference in Heterogenous Networks", 2012, pp. 1-5.

[4] J. H. Conway, R. H. Hardin and N. J. A. Sloane, "Packing Lines, Planes, etc., Packings in Grassmannian Spaces, Experimental Mathematics", Vol. 5 (1996), 139-159

[5] A Family of Optimal Packings in Grassmannian Manifolds P. W. Shor and N. J. A. Sloane, J. Algebraic Combinatorics, 7 (1998), pp. 157-163

[6] N. J. A. Sloane "How to Pack Lines, Planes, 3-Spaces, etc", Weblink: http://www2.research.att.com/~njas/grass/

[7] "QAM Theoretical BER", Weblink: http://www.raymaps.com/index.php/qam-theoretical-ber/